Contextual objectivity : a realistic interpretation of quantum mechanics.

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An attempt is made to formulate quantum mechanics (QM) in physical rather than in mathematical terms. It is argued that the appropriate conceptual framework for QM is "contextual objectivity", which includes an objective definition of the quantum state. This point of view sheds new light on topics such as the reduction postulate and the quantum measurement process.

I. INTRODUCTION

We attempt here to define a physical conceptual framework adapted to the interpretation of the usual formalism of quantum mechanics. We do not make any changes to the mathematics of this formalism, neither do we introduce new equations. Our aim is rather to present a physical interpretation of the formalism in such a way that the so-called "difficulties" of QM, such as the reduction postulate, or the measurement process, can be seen from another side. Our approach is different from the one using "consistent (or decoherent) histories"; it carefully avoids "multiple worlds", and it has nothing to do with any revival of "hidden variable theories". It should be better to consider it as a "neo-copenhagian" point of view, in the sense that it formalizes a distinction between the classical and quantum worlds. However, contrary to the copenhagian dogma, a central point in our approach will be to give an "objective reality" to the quantum state of a physical system, in a sense which is developed below.

II. A PHYSICAL DEFINITION OF THE QUANTUM STATE

In their celebrated paper written in 1935, Einstein, Podolsky, and Rosen made the following statement [1]: "If without in any way disturbing a system we can predict with certainty (i.e., with a probability equal to unity) the values of a set of physical quantities, then objective reality can be attached to this set of physical quantities". It is known that this rather "metaphysical" definition resulted in a lot of questioning and arguing. However, this will be our starting point. We will slightly modify this statement in order to give our definition of the quantum state[1] as follows.

[1]Throughout this paper "state" means "pure state". Mixed states, when needed, will be called "statistical mixtures" [2].

The quantum state of a physical system is defined by the values of a set of physical quantities, which can be predicted with certainty and measured repeatedly without perturbing in any way the system. This set of quantities must be complete in the sense that the value of any other quantity which satisfies the same criteria is a function of these values.

We will first make some comments about this definition, then draw some consequences from it

Comment 1 : The definition is clearly in agreement with the usual formalism of QM, as it can be seen when using the notion of "complete set of commuting observables" (CSCO) [2]. A quantum state is specified by the ensemble of eigenvalues corresponding to a CSCO, which can obviously be measured repeatedly without perturbing in any way the system. Actually, any physical quantity that satisfies the definition can be expressed as a function of the CSCO observables. This "completeness" condition is essential to warrant a unique correspondence between the state and the set [3].

Comment 2 : Since we consider the usual QM formalism as said above, the quantum state is mathematically described by a vector in an Hilbert space, and it evolves in time according to Schroedinger's equation. It it clear that unitary evolution from Schroedinger's equation transforms a state which satisfies our definition into a similar state, associated with a different set of physical quantities, corresponding to new well-defined measurements (that may however be not easy to perform).

Comment 3 : As an example, a coherent state $|\alpha\rangle$ of the harmonic oscillator can be in a deterministic way displaced by $(-\alpha)$ [2]. After that operation, a measurement of the photon number $\hat{n}$ gives $n = 0$ with probability one. A similar reasoning applies to an evolving gaussian wave packet: if we catch it in an appropriate harmonic potential, and we measure the energy, we find the ground state energy with probability one. Generally speaking, our definition simply uses the fact that a pure state is always a joint eigenstate of a set of commuting operators.

Comment 4 : An obvious but fundamental point is that given a quantum state, not all possible physical quantities can be predicted with certainty, but only a subset of them. That the subset is the largest possible set of independant quantities is just the definition of a CSCO. In the definition, this appears in the statement "measured repeatedly without perturbing in any way the system" : if the subset is not a CSCO, a measurement will generally change the state. How to deal with physical observables which do not commute with those of the CSCO is a crucial point, which will be discussed in section III.

Comment 5 : In order to appreciate the difference between our definition and the EPR reasoning, one should consider what "can be predicted with certainty" in a two-particle EPR state, such as a singlet state. In [1] EPR were considering measurements on separated subsystems, where the *conditional* probability of a measurement result on one particle, given a measurement done on the other particle, may take the value one. However, the *overall* result of both measurements remains inherently uncertain, simply because measurements on separated subsystems are not part of the CSCO defining the EPR state (see also section III). What actually gives a certain and reproducible result is a joint "Bell measurement" on the pair of particles [4]. An EPR state thus perfectly fits within our definition. Moreover, we may define quantum non-separability as the impossibility to define the quantum state (according to the definition given above) for a sub-part of a system which is globally in a spatially extended entangled state.

Consequence 1 : Though this may sound metaphysical, our definition implies that some "objectivity" can be attached to the quantum state, in a sense which is remarkably close to the one used by Einstein, Podolsky and Rosen. In particular, a quantum state as defined above is clearly independent of the observer. Also, it is well known from statistical physics or quantum information theory that a pure state has zero entropy. This clearly fits with a set of perfectly predictable and observer-independent properties.

Consequence 2 : Our definition can be given a meaning only by assuming that the sentence "predicted with certainty and measured repeatedly" itself has a meaning, i.e., that we are in a context where this predicate corresponds to a well defined action (i.e. a possible experiment). This is why we call our point of view "contextual objectivity": the quantum state does have an objective existence, but its definition is **inferred** from observations which are made **at the macroscopic level**. We point out that there is no need to refer to "observer's consciousness" or anything like that: all what we need is simply the the usual classical world, as the place where the measurements are made, and where their results can be recorded by any (conscious or unconscious) observer.

Consequence 3 : According to the above observations, the state of the system is conditioned by the external classical world, which, from now on, we will call the "environment". Therefore, we can use an inference principle: from an appropriate observation in the environment, one can infer the quantum state of a system. This point is crucial in what follows, because it is for instance in contradistinction with a "multiple world" point of view: in our approach, there is only one environment, and the "reality" does not only develop upwards by "ramification" of the quantum state, but also downwards by inferring microscopic states from macroscopic observations.

Consequence 4 : A fundamental consequence of all the above is that what we call the quantum state **cannot** involve different "statuses" of the environment. Different statuses of the environment can occur only if they are associated with different quantum states. This may include the case where the status of the environment is not known, and is described by using classical statistics. This corresponds to the usual "statistical mixtures" in the density matrix formalism. It should be pointed out that a statistical mixture, *contrary to a pure state*, is "contingent" (*i.e.* observer-dependant). In other terms, a statistical mixture is always associated to a (classical) missing information, which may be known by somebody else. On the other hand, nobody can know more about a pure state, than what is given in its definition [5].

We will argue below that within the framework defined above, there is no need to add a "measurement postulate": this postulate is already implied by the very definition of the quantum state.

III. MEASUREMENTS ON A QUANTUM SYSTEM

We now come to the measurement of a physical quantity which does not pertain to the set allowed by the definition of the state (*i.e.* the CSCO), and therefore which cannot be predicted with certainty. In a first approach, we will adopt the usual "decoherence" point of view [4], which is basically an attempt to calculate what is going on during the measurement, using the initial quantum state and Schroedinger's equation. Then we will criticize this approach and introduce our point of view.

In the decoherence approach, the initial steps of the measurement of a physical quantity for a system S1 can be described by using Schroedinger's equation, as the interaction of two (or several) systems S1, S2, ... As a result of this interaction, S1 is generally no longer in a well defined state, i.e. no physical quantity relevant to S1 only is predictable with certainty. The set of fully predictable physical quantities, which still exists by definition of the state, will rather correspond to joint measurements of S1 and all the other systems it has interacted with.

Remaining within the decoherence approach, the evolution of the "growing" system will at some point involve different possible values of environment variables, such as the position of a needle. Then several things happens. First, the "fully predictable quantities" which are still attached to the global state become useless in practice, because the corresponding joint measurements become unpracticable. Second, the calculation can be pushed further by concentrating on the system itself. This leads eventually to different possible quantum states for the system, each of which is associated with a macroscopically different environment, and occurs with a probability which is given by the calculation. There are several ways to look at these alternative possibilities:

(A) one may postulate that after the measurement the system is "projected" onto one possibility only, corresponding to the observed environment (usual "reduction of the wave packet" postulate)

(B) one may assume that all possibilities keep on going, but that there are continuous "branching" of universes, so that we can see one branch only, i.e. the one where we live ("many-worlds" interpretation)

(C) one may assume that the alternative possibilities describe a statistical ensemble of identically prepared systems. In accordance with the usual statistical approach, there is some "missing information" about the state of the system, corresponding to the fact that the density matrix of the system alone is a statistical mixture. This information is supposed to be written in the environment.

We list below some difficulties associated with these different points of views, then we draw a conclusion.

(i) The point of view (A) has the problem of "terminating" the unitary evolution of the state (according to Schroedinger's equation) by an "extra" non-unitary process (a projection). The question is then how to relate these two radically different types of evolution.

(ii) The point of view (B) does not explain why only one result is observed in any given experiment: in (A), the uniqueness of the result is postulated, in (B) all possible results are still there, and in (C) this question is ignored and considered to be irrelevant. The usual answer in (B), which says that only the branch "where I am" actually matters, is very puzzling, because the existence of other branches "where I am not" looks more as an artifact than a description of the physical world.

(iii) In the point of view (C), that we believe to be the most widely accepted at present, the existence of a missing information written in the environment is postulated rather than demonstrated. Actually, it can be argued that the choice of one (and only one) measurement result "must happen" at some point; then (C) seems to require an implicit use of either point of view (A) or (B).

More generally, all that can be done by the decoherence point of view is to create a "growing quantum state" as more and more sub-systems are coupled to the initial system S1. As far as we can see, this cannot warrant that a classical- looking environment will be recovered at the end. On the other hand, one must remember our initial argument that the environment was actually a prerequisite for the definition of the quantum state.

We are thus lead to our main conclusion, which is simply that **the uniqueness of the measurement's result is guaranteed by the uniqueness of the environment, which is actually, as it was said from the beginning, a necessary condition for the very definition of the quantum state**.

Comment 1 : This may hurt the intuitive feeling that the measurement should either "reveal" a pre-existing state of S1 (naive "classical" view), or "drive" S1 from an initial to a final state, including the environment (naive "quantum" view). Actually, for quantities which are not in the initial CSCO, there is in general **no** pre-existing quantum state (i.e. well-defined properties) of S1, to be revealed at the end of the measurement. On the other hand, S1 is not "driven" somewhere: according to the QM formalism, S1 has become intricated with the entire environment, and it is only as a consequence of the procedure which allows one to define a quantum state, that such a state is eventually observed. The "contextual" essence of QM lies in the compatibility between the "downstream" definition of a quantum state, and the "upstream" evolution given by the decoherence calculation.

Comment 2 : In our approach, **the boundary between quantum and classical mechanics results from the fact that the quantum state is defined in terms of the observed environment, and thus cannot include this environment**. If the evolution of the system leads to an "entanglement with the environment", then the global state becomes irrelevant (because the corresponding fully predictable observables are out of reach), and the system's state is no longer defined. It appears then that the system is in a statistical mixture of states, where the "missing information" can be read out in the environment. The decoherence approach allows one to calculate the corresponding diagonal density matrix, but it is unable to "terminate" the calculation, by extracting only one result. The "interpretative" step that is required at this point is more than a matter of convenience related to the fact that the global state is too complicated to be followed. Our claim is that *this interpretative step is required from the beginning*, because it is precisely the one which is required in order to define the quantum state (see section II). In practice, our conclusion agrees quite well with the point of view (C) introduced above, which is now built inside our very definition of the quantum state, and does not require an implicit use of either (A) or (B).

Comment 3 : A well known question is then: how to separate the "system" from the "environment" ? Actually, defining the environment is not ambiguous, since it is the point from which the quantum state is defined (i.e. the classical world where we are). The definition of the system is more subtle, and it depends on the precise physical system which is studied. The existence of this (very flexible) boundary, far from being a weakness of QM, is actually a strength. In particular, this opens the way to "mesoscopic quantum states", which are one of the main challenges of today's quantum mechanics.

Comment 4 : Our view is consistent with the idea that QM is a non- deterministic theory. Though the evolution of the quantum state is deterministic, non-determinism is due to the fact that in general (*i.e.* each time a non-CSCO observable is measured) the measurement redefines the state, taking into account the contextual nature of the theory. Similarly, our approach does not change anything to quantum non-separability: a quantum state is generally non-separable, and may violate Bell's inequalities. Despite our initial use of the EPR wording, our point of view thus differs from EPR on two central issues:

(i) not all physical quantities are fully predictable, only a subset of them (Heisenberg's inequalities)

(ii) the quantum state has an objective reality, but it is non-separable (Bell's inequalities).

Non-separability is probably the most original feature of quantum mechanics, and directly contradicts "local realism", which is the view expressed mathematically in the hypothesis leading to Bell's inequalities. On the other hand, there is no contradiction with relativistic causality, and not even with "naive" realism, according to our definition of the quantum state.

Comment 5 : Do we add anything new with respect to the "reduction of the wave packet postulate" ? Again, the difference is that this postulate is no longer necessary, but is built inside our definition of the quantum state: from the beginning, a quantum state is (objectively!) defined with respect to observations which are made within the environment. It is just a consequence of this definition that new observations in a changing environment lead to a new - but again uniquely defined - quantum state.

Comment 6 : Clearly our very definition of a quantum state relies on something external to it, as was often advocated by Bohr. However, we do not accept to infer that a quantum state is merely "subjective knowledge". We consider that our definition of the quantum state is objective, in a sense that has been discussed above. In our view, *a quantum state gives the best possible objective description of a suitably isolated subsystem of the physical world* (what is a suitable isolation is defined by QM as being "decoherence-free"). On the other hand, we note that our point of view makes it very difficult to speak about anything like a "quantum state of the universe".

Comment 7 : Most interpretations of QM, including the present one, agree on the fact that the quantum structure of the microscopic world explains the features of the classical structure of the macroscopic world. But another question is in which sense it is possible to "construct" or "deduce" classical physics from quantum physics. There are at least two answers to this question. The first one is simply to say that the purpose of QM is to calculate correlation functions between successive measurement events. This is a very "minimalistic" approach from a conceptual point of view, but it is extremely useful both for teaching and for using QM. Our approach is fully compatible with this "correlation function" approach: in some sense, we are only trying to put words around it, in a way as consistent as possible. Another answer is to say that the classical world can be explicitly deduced from QM by using *e.g.* coarse graining and decoherent histories. However, this point of view leads also to the idea of the "wave function of the universe", including an infinite infinity of branches corresponding to all possible results of all possible "measurements". We are clearly very doubtful about these approaches, precisely because the formulation of QM is contextual, that is, depends at its start from what it should "explain" at its end, i.e., the observed environment. As said above, our "contextual objectivity" point of view is that QM is the best possible objective description of a "suitably isolated" (or "decoherence-free") **subsystem**. This is already a lot in terms of predictive power, but if carried out of this realm, QM does turn into a "smoky dragon", which tries in vain to capture its origin.

IV. CONCLUSION

With the general goal to provide a "status report" rather than any final argument, we have presented an attempt to reconcile the still prevailing Copenhagian point of view about QM, with the more modern "decoherence" points of view. A brief summary of the main our main statements is presented here :

(1) Measurements allow one to define the quantum state of a physical system in an objective and observer-independant way (the new point here is that we apply this "classical" statement to QM as well).

(2) Given a quantum state, only a *subset* of all physical quantities is fully predictable (on the other hand, given a classical state, *all* physical quantities can be known; denying this possibility *does not* impede our objective definition of the quantum state).

(3) A quantum state may be non-separable : the physical properties of a sub-system are then among the non-predictable ones, even though the state of the whole system is perfecty defined (on the other hand, the classical state of a compound system can always be defined by cutting it in parts, and defining the state of each part; again, denying this possibility *does not* impede our objective definition of the quantum state).

(4) If one measures a physical property which is not among the predictable ones, the initial system becomes a (non predictable) sub-system in the measurement apparatus, and the state is reset by looking at the measurement result (this brings back to step one).

Our approach differs from the orthodox Copenhagian approach, by insisting upon the objective character of the quantum state. It also differs from the standard decoherence approaches, by insisting upon the fact that the boundary between classical and quantum mechanics cannot be "erased", since a quantum state "involving the environment" cannot be consistently defined.

Both aspects are contained in the definition of a quantum state from the values of a set of physical quantities which can be predicted with certainty and measured repeatedly without perturbing in any way a system.

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[1] A. Einstein, B. Podolsky and N. Rosen, *"Can quantum mechanical description of reality be considered complete"*, Phys. Rev. **47**, 777 (1935)

[2] C. Cohen-Tannoudji, B. Diu and F. Laloe, "Mécanique Quantique", Hermann, 1977

[3] Jean-Louis Basdevant, private communication.

[4] We assume that the reader is familiar with these issues, which are central for quantum information theory. For an introduction see *e.g.* John Preskill, Lecture Notes on Quantum Information and Computation, Caltech (1998), or the special issue of Physics World, march 1998.

[5] In the line of the present approach, the most natural way for specifying a statistical mixture is by using the eigenvalues and eigenvectors of the density matrix. The eigenvalues clearly appear as the probabilities of different (orthogonal) states represented by the eigenvectors, as it would be the case classically. When the density matrix has degenerate eigenvalues, the eigenvectors basis is not unique. In our approach, this simply means that not enough (classical) information is available to specify the system's state.